The Electron Gas

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Electron Gas

- Metals are excellent conductors of heat and electricity.
- It is observed that electrical current in a metal involves the flow of electrons, but not ions.
- A simple model proposed by Drude in 1900 assumes that when the atoms of a metallic element are brought together to form a metal, the valence electrons become detached and are free to move within the metal lattice.
- If we assume that the electrons do not interact with one another, then they collectively must act like a monatomic ideal gas.

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Drude Energy

- If free electrons do act like ideal gas, each one would contribute 3/2 kT to the total energy, so

$$E = N_0 \frac{3}{2} kT$$

- The specific heat is then

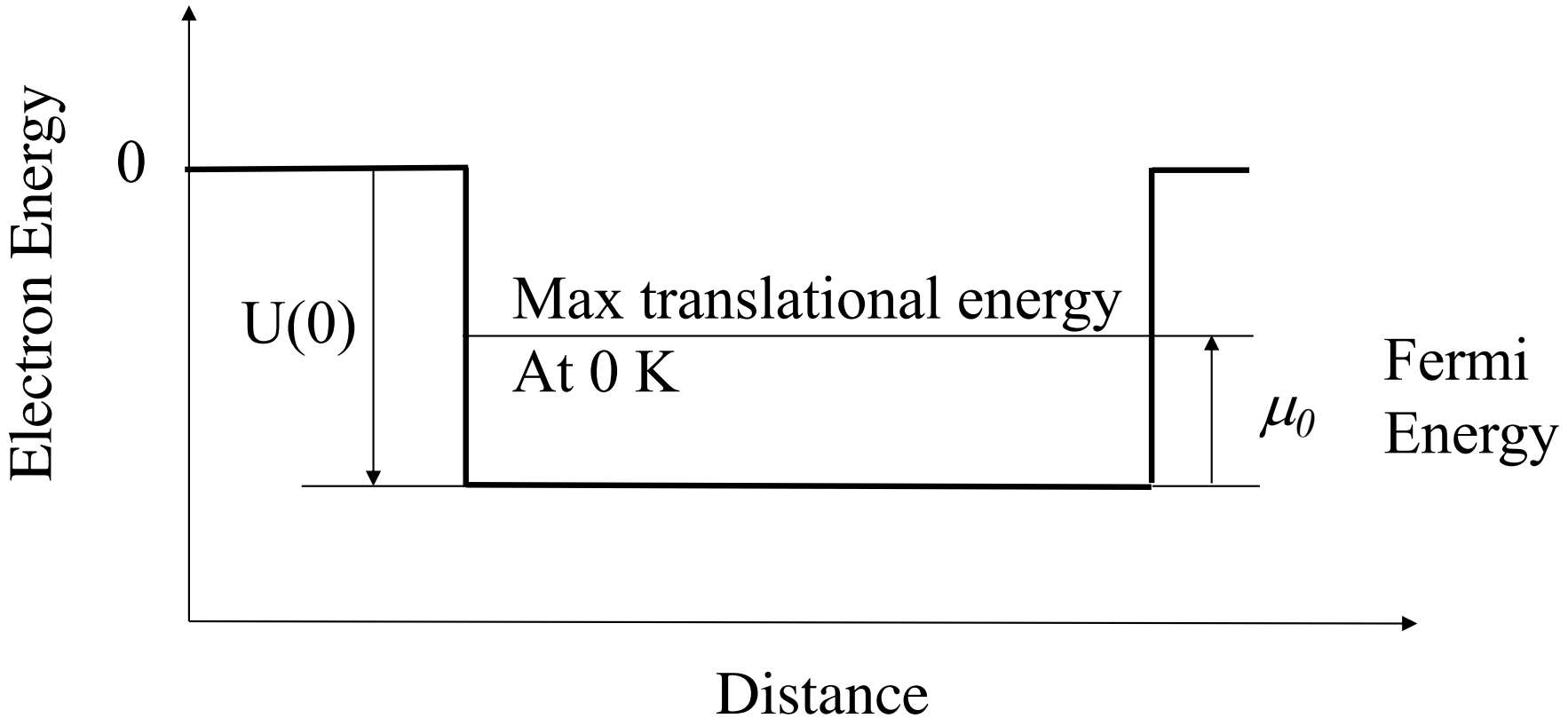
$$c_v = \frac{3}{2} k$$

- However, this doesn't work very well and a full quantum analysis is required.

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Electron Energies

Since electrons satisfy the exclusion principle, they will fill the lattice in increasing energy order.



The maximum electron energy at 0K is called the Fermi energy

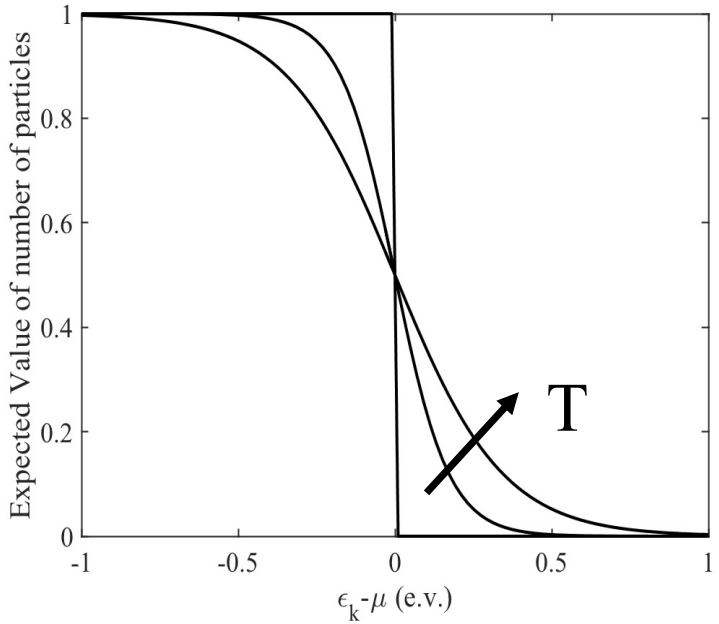
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Number Distribution

- The quantum state number distribution for a Fermion is

$$N_k = \frac{1}{e^{(\varepsilon_k - \mu)/kT} + 1}$$

- In the continuum limit the single electron energy distribution is



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Fermi Energy

- Integrating the energy distribution at 0K

$$N = 4\pi \frac{8\pi}{3}\left(\frac{2m}{h^2}\right)^{3/2} V\int_0^{\mu_0} \varepsilon^{1/2} d\varepsilon = \left(\frac{2m}{h^2}\right)^{3/2} V\left(\mu_0\right)^{3/2}$$

- Thus

$$\mu_0 = \frac{h^2}{2m}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{N}{V}\right)^{2/3}$$

- where N/V is the free electron number density.
- Values are typically 1-5 ev.

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Number Density of Electrons

- The number density of free electrons depends on the specific metallic atom.
- The table on the next slide shows experimentally determined values.
- However, for many metals, there is about one free valence electron per atom.

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Free Electron Densities

Table 1.1
FREE ELECTRON DENSITIES OF SELECTED METALLIC ELEMENTS[a]

ELEMENT	Z	n (10^{22}/cm^3)	r_s(Å)	r_s/a_0
Li (78 K)	1	4.70	1.72	3.25
Na (5 K)	1	2.65	2.08	3.93
K (5 K)	1	1.40	2.57	4.86
Rb (5 K)	1	1.15	2.75	5.20
Cs (5 K)	1	0.91	2.98	5.62
Cu	1	8.47	1.41	2.67
Ag	1	5.86	1.60	3.02
Au	1	5.90	1.59	3.01
Be	2	24.7	0.99	1.87
Mg	2	8.61	1.41	2.66
Ca	2	4.61	1.73	3.27
Sr	2	3.55	1.89	3.57
Ba	2	3.15	1.96	3.71
Nb	1	5.56	1.63	3.07
Fe	2	17.0	1.12	2.12
Mn (α)	2	16.5	1.13	2.14
Zn	2	13.2	1.22	2.30
Cd	2	9.27	1.37	2.59
Hg (78 K)	2	8.65	1.40	2.65
Al	3	18.1	1.10	2.07
Ga	3	15.4	1.16	2.19
In	3	11.5	1.27	2.41
Tl	3	10.5	1.31	2.48
Sn	4	14.8	1.17	2.22
Pb	4	13.2	1.22	2.30
Bi	5	14.1	1.19	2.25
Sb	5	16.5	1.13	2.14

[a] At room temperature (about 300 K) and atmospheric pressure, unless otherwise noted. The radius r_s of the free electron sphere is defined in Eq. (1.2). We have arbitrarily selected one value of Z for those elements that display more than one chemical valence. The Drude model gives no theoretical basis for the choice. Values of n are based on data from R. W. G. Wyckoff, *Crystal Structures*, 2nd ed., Interscience, New York, 1963.

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Fermi Temperature

- Defined as

$$T_F = \frac{\mu_0}{k}$$

- Since 1 ev = 11,600 K, the Fermi temperatures are typically very large.
- Thus, electron gases behave as though they are at very low temperatures.

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Determining the Fundamental Relation

- At 0K this is easy. It is the ideal gas result (Helmholtz Function)

$$A_0 = NU(0) + \frac{3}{40} N \frac{h^2}{m} \left(\frac{3N}{\pi V} \right)^{2/3}$$

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Higher Temperatures

- Calculations would be simplified if the Maxwell-Boltzmann limit were satisfied. However

$$\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\frac{V}{N} \approx 1.42x10^{-3} << 1$$

- for copper at its normal melting point. Therefore, the full Fermi-Dirac treatment must be followed.

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Fundamental Relation

- Thus, series methods must be used

$$A = A_0 \left[1 - \frac{5\pi^2}{12} \left(\frac{T}{T_F} \right)^2 + \cdots \right]$$

$$U = U_0 \left[1 + \frac{5\pi^2}{12} \left(\frac{T}{T_F} \right)^2 - \cdots \right]$$

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In all cases $T_F >> T$, so that

$$c_v \cong \frac{\pi^2}{2} k \frac{T}{T_F}$$

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